Scaling law of plasma turbulence with nonconservative fluxes

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It is shown that in the presence of anisotropic kinetic dissipation existence of the scale invariant power law spectrum of plasma turbulence is possible. The obtained scale invariant spectrum is not associated with the constant flux of any physical quantity. Application of the model to the high frequency part of the solar wind turbulence is discussed.

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Dissipation range of incompressible hydrodynamic turbulence has been extensively studied by different authors [1–9] due to the fact that smallest scale perturbations display strong intermittency, even at Reynolds numbers so low that there is no basis for fractal cascade. The kinetic energy spectrum $E(k)$ of the hydrodynamic turbulence in the far dissipation range behaves as

$$E(k) \sim k^{\alpha_1} \exp[-\alpha_2 (k/k_d)^n], \tag{1}$$

where k_d is Kolmogorov dissipation wave number, α_1 and α_2 are constants and $1 \leqslant n \leqslant 2$ (see, e.g., [1] and references therein).

In the case of plasma turbulence existence of various kinetic mechanisms of dissipation makes the situation much more complicated. For instance, observations of the solar wind turbulence [10–12] strongly suggest steep power law spectrum of the magnetic field fluctuations for the frequencies higher then ion cyclotron frequency for which kinetic mechanisms of dissipation are dominant. In contrast with viscosity, kinetic dissipation of plasma waves in the presence of background magnetic field is usually strongly anisotropic. In the presented paper we show that in the presence of (i) anisotropic kinetic dissipation; and (ii) if the nonlinear transfer is governed by the scattering of the plasma waves by low frequency waves, then one should expect the scale invariant power law spectrum of the plasma turbulence. It should be emphasized that the scale invariant spectrum is not associated with the constant flux of any physical quantity due to the presence of kinetic dissipation.

The general equation that governs the evolution of any averaged characteristic Z of the homogenous turbulence which is conserved by nonlinear interactions (in the case of hydrodynamic turbulence Z is usually associated with energy density) in the wave number space has the form

$$\frac{\partial Z}{\partial t} = J + D + S, \tag{2}$$

where S and D describes the source and dissipation of Z and J accounts for the accumulation of Z due to the nonlinear interactions among the various wave number components of the turbulent field, such as velocity and magnetic fields. If the nonlinear transfer term J serves only to redistribute Z and does not change the total amount, then one can define the flux field F in the wave number space [13]

$$J = -\nabla \cdot \mathbf{F}(\mathbf{k}), \tag{3}$$

so the property of conservation is automatically fulfilled.

Further we assume that nonlinear interactions are local in the sense that the most contributions to $\mathbf{F}(\mathbf{k})$ are from nearby regions of the $\mathbf{k}$ space. We make diffusion approximation to the wave number space transport, i.e., we assume that the flux can be presented as

$$F_i(\mathbf{k}) = -D_{ij}\frac{\partial Q}{\partial k_j}, \tag{4}$$

where D_{ij} are diffusion coefficients and Q is potential, that in the general case is some function of $\mathbf{k}$ and Z. So in the inertial range Eq. (2) takes the form

$$\frac{\partial Z}{\partial t} = \frac{\partial}{\partial k_i}\left[D_{ij}(k,Z)\frac{\partial Q(k,Z)}{\partial k_j}\right]. \tag{5}$$

The diffusion approximation for isotropic hydrodynamic turbulence was first introduced by Leith [13]. Afterwards the same concept was successfully applied to the plasma turbulence both in the strong [14] and weak [15] turbulence regimes. For isotropic hydrodynamic turbulence Z corresponds to energy density $\mathcal{E}(\mathbf{k})$, $D_{ij} \sim k^{9/2}\delta_{ij}$, and $Q \sim \mathcal{E}^{3/2}$ [13], where δ_{ij} is kronecker delta. Combining Eqs. (2)–(4) one can readily obtain the famous Kolmogorov spectrum $\mathcal{E} \sim k^{-11/3}$ for the inertial range of the hydrodynamic turbulence. In the case of the plasma turbulence the situation is more complicated. Existence of the background magnetic field usually leads to the anisotropy of the nonlinear cascade.

Consider plasma turbulence in some frequency range where there is no source of the turbulence ($S=0$) and there exists kinetic dissipation of plasma waves. As it was shown in Refs. [2,3], energy transfer in the dissipation range of hydrodynamic turbulence is dominated by nonlocal triads in which one leg is in the energy-containing (low-k) range. Similarly, we suppose that the strongest nonlinear interaction is the scattering of the high frequency waves by low frequency ones from the inertial range of the plasma turbulence. In the weak turbulence theory [16] this process requires fulfillment of the resonant conditions

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$$\mathbf{k}_1 = \mathbf{k}_2 + \mathbf{K}, \quad \omega_1 = \omega_2 + \Omega, \tag{6}$$

where $\mathbf{k}_{1,2}$, $\omega_{1,2}$, and $\mathbf{K}$, Ω are wave numbers and frequencies of high and low frequency waves, respectively. Due to the fact that $\omega \gg \Omega$ and $k_{1,2} \gg K$, the change of the wave number is small in the unit act of the scattering ($|\mathbf{k}_1 - \mathbf{k}_2| \ll |k_{1,2}|$) and therefore diffusion approximation is applicable (see, e.g., [15,37]).

We incorporate the dissipation in the model as follows: we assume that the dissipation of the waves is negligible compared to the nonlinear interaction if the angle of the propagation with respect to the external magnetic field θ is less than some angle θ_0, whereas the opposite limiting case takes place for $\theta > \theta_0$. This model seems reasonable for the transverse waves in collisionless plasma, such as whistler waves and electromagnetic (ordinary and extraordinary) waves in electron-positron plasma [17,18]. Indeed, in this case the main mechanisms of dissipation are Landau and cyclotron damping. Both Landau and multiple cyclotron resonances do not affect the transverse waves for parallel propagation with respect to the background magnetic field (see, e.g., [19]), whereas for relatively large angles of propagation both mechanisms can be on work. In the case of Landau damping this is caused by the fact that a ambient propagating wave has a nonzero electric field component parallel to the background magnetic field—a necessary condition of Landau damping.

The second assumption of the presented model, that the nonlinear transfer is governed by the scattering of plasma waves by low frequency ones, is also satisfied for both kinds of above mentioned wave modes. For of whistler waves in the solar wind, as it is shown below the strongest nonlinear process is the scattering of whistler waves by low frequency magnetohydrodynamic waves from inertial range of the solar wind turbulence. Similarly, the strongest nonlinear process that governs evolution of electromagnetic waves in electron positron plasma is their scattering by low frequency Langmuir waves (see e.g., [20] and references therein).

It should be noted that present model is not valid for Alfvénic turbulence. Alfvén waves have low frequency compared to the ion cyclotron frequency and therefore they are not affected by cyclotron damping. Landau damping of Alfvén waves also have unusual properties [21] incompatible with the presented model. Additionally, the second assumption about nonlinear transfer is also violated in this case (see, e.g., [22]).

Note, that if θ_0 is not extremely small, presented model implies that the wave with $\theta=0$ should take part in many scattering events before it can be transferred to the dissipation area. This circumstance allows us to use Eq. (5) for the conical area in the wave number space $\theta < \theta_0$, and take the dissipation into account by requesting $Q(\mathbf{k})$ to vanish at $\theta = \theta_0$.

In the case of Alfvénic turbulence numerical simulations [23] as well as an analysis of three and four wave resonant conditions provide that the turbulent cascade is strongly anisotropic. In the weak turbulent regime there is no cascade in the direction parallel to the background magnetic field at all [24]. When the dispersion of both high and low frequency waves can be considered as nearly isotropic for $\theta < \theta_0$, then one can expect different diffusion coefficients for the directions parallel and perpendicular to the wave vector $\mathbf{k}$ of the high frequency waves. We consider scale invariant diffusion, i.e., assume $D_{ij}(k,Z) = d_{ij}\delta_{ij}k^{\alpha_i}$, and $Q \sim k^{\beta_1}Z^{\gamma_1} \equiv k^{\beta_2}E(k)^{\gamma_2}$ where d_{ij} are not functions of k and Z.

With these assumptions Eq. (5) reduces to the following:

$$\frac{1}{k^2}\frac{d}{dk}\left(k^2 D_\parallel \frac{dQ}{dk}\right) + \frac{D_\perp}{k^2 \sin\theta}\frac{d}{d\theta}\left(\sin\theta \frac{dQ}{d\theta}\right) = 0, \tag{7}$$

with boundary condition $Q(k,\theta_0)=0$. Here $D_\parallel = d_\parallel k^{\alpha_\parallel}$ and $D_\perp = d_\perp k^{\alpha_\perp} k_s^{\alpha_\parallel - \alpha_\perp}$, and k_s denotes the minimal wave number for which the formulated model is valid. Let us first consider the case when parallel and perpendicular diffusion coefficients have the same scaling law, i.e., $\alpha_\parallel = \alpha_\perp \equiv \alpha$. Using standard methods of variable separation, i.e., representing potential as $Q(k,\theta) = R(k)\Psi(\theta)$ we obtain for the solution of Eq. (7)

$$Q = \sum_{m=1}^{\infty} B_m k^{-c_m} P_{\nu_m}(\cos\theta), \tag{8}$$

where coefficients B_m are determined by the source of the turbulence at small wave numbers $k=k_s$, $P_{\nu_m}(\cos\theta)$ are Legendre functions of the first kind, and ν_m are the solutions of the eigenvalue problem $P_\nu(\cos\theta_0)=0$, arranged in order of increasing magnitude and

$$c_m = \frac{1}{2}\left[-\alpha - 1 - \sqrt{(\alpha+1)^2 + 4\nu_m^2 \frac{d_\perp}{d_\parallel}}\right]. \tag{9}$$

If $\theta_0 < \pi/2$ the value of the first eigenvalue ν_1 can be approximated as [25]

$$\nu_1 \approx \frac{2.405}{\theta_0} - \frac{1}{2}. \tag{10}$$

For high wave numbers ($k \gg k_s$) the leading term is

$$Q \approx B_1 k^{-c_1} P_{\nu_1}(\cos\theta). \tag{11}$$

Note that without kinetic dissipation isotropic solutions of Eq. (7) is $Q \sim k^{-\alpha-1}[E(k) \sim k^{-(\alpha+1+\beta_2)/\gamma_2}]$, which correspond to constant flux of Z. In contrary, obtained scale invariant asymptotic solution [see, Eq. (11) is not associated with constant flux of any physical quantity, due to the presence of kinetic dissipation. Alternatively, in contrast with hydrodynamic turbulence, where the spectrum in the dissipation range is exponential, obtained results show that the energy spectrum should decrease as a power law in the dissipation range if the diffusion coefficients have the same scaling law.

Now consider the case $\alpha_\perp - \alpha_\parallel \equiv \Delta\alpha \neq 0$. Using the same technique of variable separation and introducing new variables $K = k/k_s$ and $P(K) = k^{1+\alpha_\parallel/2}R(K)$, Eq. (7) yields

$$K^2 \frac{d^2 P}{dK^2} - \left[\frac{\alpha_\parallel(\alpha_\parallel+2)}{4} + \nu_m^2 \frac{d_\perp}{d_\parallel} K^{\Delta\alpha}\right] P = 0. \tag{12}$$

If $\Delta\alpha < 0$, for $K \gg 1$ one can drop the second term in the squire brackets. This yields the result that coincides with the result of the isotropic case $Q \sim k^{-\alpha_\parallel - 1}$, i.e., asymptotically dissipation has no influence on the cascade.

On the other hand, when $\Delta\alpha > 0$, for $K \gg 1$ one can drop the first term in the squire brackets. Obtained equations can be solved in terms of modified Bessel functions. Using asymptotic properties of modified Bessel functions [25] we obtain

$$Q \sim K^{-(\alpha_\parallel+1)/2-\Delta\alpha/4} \exp\left(-\frac{2\nu_1}{\Delta\alpha_\parallel}\sqrt{\frac{d_\perp}{d_\parallel}}K^{\Delta\alpha_\parallel/2}\right), \qquad (13)$$

Consequently, in the case under consideration the spectrum is exponential. Although it should be noted, that the decay is more soft than in the hydrodynamic turbulence when $\Delta\alpha$ is relatively small ($\Delta\alpha < 2$).

One of the possible applications of the presented model is the high frequency part of the solar wind turbulence spectrum. Various spacecraft observations show the presence of persistent magnetic fluctuations in the solar wind over a broad range of frequencies [10–12]. For low frequencies ($f \lesssim 10^{-2} - 10^{-3}$ Hz) the magnetic field spectrum vary as approximately $E_M(f) \sim f^{-1}$. For higher frequencies, up to proton cyclotron frequency ($f \sim 0.1 - 1$ Hz), the Kolmogorov spectrum $f^{-5/3}$ is observed. This is believed to be the inertial interval of the solar wind turbulence. The change of slope and rapid decrease in the intensity near the ion cyclotron frequency is usually considered to be due to the absorption of Alfvén waves by ion cyclotron damping or Landau damping [26]. At the frequencies, higher than the ion cyclotron frequency, weak but persistent levels of magnetic fluctuations, that can be well approximated by the power law spectrum f^{-3}, are observed up to the electron cyclotron frequency. These fluctuations are usually associated with the whistler waves [11]. The nature of this high frequency part of the spectrum remains unexplained.

Whistler turbulence have been intensively studied by different authors both in strong [27–29] and weak [30,31] turbulent regimes. If one assumes the existence of the inertial interval of the whistler turbulence, then the Kolmogorov-type dimensional analysis yields for the magnetic spectrum [28] $E_M(k) \sim k^{-7/3}$, that is incompatible with observations [note that due to the relation $E_M(f)df \sim E_M(k)dk$, and taking into account Doppler shift and dispersion of whistler waves $f \sim k^2$, observed f^{-3} spectrum corresponds to k^{-v} with $v \sim 5 - 6$ in the wave number space].

There exist several different directions of the research for an explanation of the high frequency solar wind spectrum. The first approach [27,32] is based on the fact that governing equations of Hall magnetohydrodynamics besides energy, conserves two other second order (with respect to the field variables) quantities—magnetic and generalized helicity [33]. Therefore, stationary Kolmogorov-type spectrum can be "driven" not only by energy cascade, but also by the cascade of magnetic and generalized helicities [27]. In Ref. [34] short wavelength dispersive properties of the magnetosonic-whistler waves have been studied as a possible reason for the spectrum steepening. An alternative approach to the explanation of the high frequency magnetic fluctuations spectrum in the solar wind implies incorporation of the linear kinetic effects, such as Landau and cyclotron damping. It has been shown [35] that simple incorporation of dissipation term to the energy budget equation leads to a sharp cutoff of the energy spectrum. On the other hand, total ignorance of dissipation leads to a much more smooth spectrum than compared to the observed one.

The model considered in the presented paper could have important consequences for the explanation of the high frequency part of the solar wind spectrum. Whistler waves propagating along the background magnetic field are affected by neither Landau nor cyclotron damping. Based on the numerical solution of linear Vlasov equation [35] the angle θ_0 at which kinetic dissipation becomes dominant can be estimated as $\theta_0 \sim \pi/6$. The level of whistler-wave fluctuations is low in the sense that $\langle b_w^2 \rangle / B_0^2 \ll 1$, where $\langle b_w^2 \rangle$ is the rms of the whistler-wave magnetic field fluctuations and $\mathbf{B}_0$ is the background magnetic field. Therefore, the study can be held in the framework of the weak turbulence theory. Possible nonlinear processes includes (a) four wave resonant interactions of whistler waves [it can be shown that if $\theta_0 < \pi/3$ then the three wave resonances of whistler waves are absent, i.e., with this restriction for all three waves resonant conditions similar to (6) do not have nontrivial solutions]; (b) induced scattering of whistler waves by ions; and (c) scattering of whistler waves by low frequency magnetohydrodynamic waves from the inertial range of the turbulence, i.e., three wave interactions which involve two whistlers and one magnetohydrodynamic wave. Detailed analysis of the nonlinear processes of the solar wind whistler waves will be presented elsewhere. Here we note that characteristic time scales of these processes, are respectively, proportional to $\tau_a \sim N^{-2}$, $\tau_b \sim N^{-1}$, and $\tau_c \sim 1$, where $N(\mathbf{k}) \equiv \mathcal{E}(\mathbf{k})/\omega(\mathbf{k})$ is the number density of whistler waves. Consequently, the strongest nonlinear process that should be responsible for the formation of the high frequency spectrum is the scattering of whistler waves by low frequency magnetohydrodynamic waves. This process conserves the total number of whistler waves [15], and therefore $Z \equiv N(\mathbf{k})$.

It can be shown that Alfvén waves do not interact with whistlers through three wave resonances, whereas kinetic Alfvén waves do [36]. Another possibility is the scattering of whistler waves by fast magnetosonic waves. Here we consider only the second possibility.

Analytical calculations of diffusion coefficients are very complicated even in the incompressible limit. In the present paper we perform qualitative analysis of three wave interactions of whistler and fast magnetosonic waves and determine relations between diffusion coefficients that correspond to the observed spectrum. For this purposes we use equations of incompressible Hall magnetohydrodynamics [33]

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times [(\mathbf{V} - \nabla \times \mathbf{B}) \times \mathbf{B}], \qquad (14)$$

$$\frac{\partial(\mathbf{B} + \nabla \times \mathbf{V})}{\partial t} = -\nabla \times [(\mathbf{B} - \nabla \times \mathbf{V}) \times \mathbf{V}], \qquad (15)$$

where time and space variables are measured in units of ion giroperiod ω_{ic}^{-1} and ion skin depth λ_i, respectively. Analysis of these equations shows that for the triad interaction of whistler and fast magnetosonic waves the strongest nonlinear term is the second one on the right-hand side of Eq. (14). Taking this into account and noting that the wave number of whistler waves is much greater than the wave number of fast magnetosonic waves, the Fourier transform of Eq. (14) yields

$$\frac{\partial \mathbf{b}_{\mathbf{k}}^{w}}{\partial t} - k_{\parallel}B_0(\mathbf{k}\times\mathbf{b}_{\mathbf{k}}^{w}) = i\mathbf{k}\times[\mathbf{b}^{f}\times(\boldsymbol{\nabla}\times\mathbf{b}^{w})]_{\mathbf{k}}, \qquad (16)$$

where index $\mathbf{k}$ denotes the Fourier transform and superscripts w and f indicate that corresponding values correspond to whistler and fast magnetosonic waves, respectively.

Using helicity decomposition [31] one can apply the standard technique of the weak turbulence theory developed for two types of interacting waves [15]. But we do not perform this analysis here due to the reason that the only thing that we need for further analysis is the scaling index of the matrix element of interaction T that immediately follows from Eq. (16): $T \sim k^2$. In the framework of the weak turbulence theory the dynamics is totally determined by linear dispersions of waves and matrix elements of interaction. The method of finding the scaling index of diffusion coefficients was developed in Ref. [15], which for the case under consideration yields $\alpha=-1$ and $\beta_1=0, \gamma_1=1$. Therefore, Eq. (11) yields

$$N(\mathbf{k}) \sim k^{-c_1}, \qquad (17)$$

where $c_1=\nu_1(d_\perp/d_\parallel)^{1/2}$.

To obtain spectral index δ of corresponding energy spectrum $E(f)\sim f^{-\delta}$, we note that

$$E(f) \sim E(k)\frac{dk}{df} \sim \mathcal{E}(\mathbf{k})k^2\frac{dk}{df} \sim N(\mathbf{k})k^3. \qquad (18)$$

Taking also into account that for whistler waves $f\sim k^2$, Eqs. (17) and (18) yield

$$\delta = \frac{c_1-3}{2}. \qquad (19)$$

As it was mentioned above, according to observations $\delta \approx 3$. Taking also $\theta_0=\pi/6$, Eqs. (10) and (19) yield $d_\perp/d_\parallel \approx 5$. Obtained results seem reasonable, due to the fact that in the magnetized media perpendicular cascade rate usually significantly exceeds parallel cascade rate (see, e.g., [38] and references therein).

In the present paper plasma turbulence in the presence of anisotropic kinetic dissipation is considered. It is shown that if the nonlinear transfer is governed by the scattering of the plasma waves by low frequency waves, then the development of a asymptotic scale invariant power law spectrum of the plasma turbulence is possible. Obtained scale invariant spectrum is not associated with the constant flux of any physical quantity due to the presence of kinetic dissipation. Corresponding spectral index is given by Eq. (9) with $m=1$. Possible application of the present model to the high frequency part of the solar wind spectrum has been analyzed.

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[1] S. Chen, G. Doolen, J. R. Herring, R. H. Kraichnan, S. Orszag, and Z. S. She, Phys. Rev. Lett. **70**, 3051 (1993).
[2] J. A. Domaradzki and R. S. Rogallo, Phys. Fluids A **2**, 413 (1990).
[3] P. K. Yeung and J. G. Brasseur, Phys. Fluids A **3**, 884 (1991).
[4] K. R. Sreenivasan and A. Bershadskii, Indian J. Phys., A **64**, 315 (2005).
[5] S. K. Dhar, A. Sain, and R. Pandit, Phys. Rev. Lett. **78**, 2964 (1997).
[6] T. Gotoh, Phys. Rev. E **57**, 2984 (1998).
[7] L. Sirovich, L. Smith, and V. Yakhot, Phys. Rev. Lett. **72**, 344 (1994).
[8] Z. S. She and E. Jackson, Phys. Fluids A **5**, 1526 (1993).
[9] D. Pisarenko, L. Biferale, D. Courvoisier, U. Frisch, and M. Vergassola, Phys. Fluids A **5**, 2533 (1993).
[10] P. J. Coleman Jr., Astrophys. J. **153**, 371 (1968).
[11] K. U. Denskat, H. J. Beinroth, and F. M. Neubauer, J. Geophys. **54**, 60 (1983).
[12] R. J. Leamon, C. W. Smith, N. F. Ness, W. H. Matthaeus, and H. K. Wong, J. Geophys. Res. **103**, 4775 (1998).
[13] C. E. Leith, Phys. Fluids **10**, 1409 (1967).
[14] Y. Zhow and W. H. Matthaeus, J. Geophys. Res. **95**, 14881 (1990).
[15] V. E. Zakharov and E. A. Kuznetsov, Sov. Phys. JETP **48**, 458 (1978).
[16] B. B. Kadomtsev, *Plasma Turbulence* (Academic, New York, 1965), p. 54.
[17] A. S. Volokitin, V. V. Krasnosel'skix, and G. Z. Machabeli, Fiz. Plazmy **11**, 531 (1985).
[18] J. Arons and J. J. Bernard, Astrophys. J. **302**, 120 (1986).
[19] A. F. Aleksandrov, L. S. Bogdankevich, and A. A. Rukhadze, *Principles of Plasma Electrodynamics* (Springer-Verlag, New York, 1984).
[20] G. Z. Machabeli and G. Gogoberidze, Astron. Zh. **49**, 463 (2005).
[21] D. Montgomery, J. Geophys. Res. **97**, 4309 (1992).
[22] S. Galtier, S. V. Nazarenko, A. C. Newell, and A. Pouquet, Astrophys. J. **564**, L49 (2002).
[23] J. V. Shebalin, W. H. Matthaeus, and D. Montgomery, J. Plasma Phys. **29**, 525 (1983).
[24] P. Goldreich and S. Sridhar, Astrophys. J. **485**, 680 (1997).
[25] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions* (Dover Publications, Inc., New York, 1965).
[26] R. J. Leamon, C. W. Smith, N. F. Ness, and H. K. Wong, J. Geophys. Res. **104**, 22331 (1999).
[27] V. Krishan and S. M. Mahajan, Nonlinear Processes Geophys. **12**, 75 (2005).
[28] D. Biskamp, E. Schwarz, and J. F. Drake, Phys. Rev. Lett. **76**, 1264 (1996).
[29] J. Cho and A. Lazarian, Astrophys. J. **615**, L41 (2004).

[30] V. N. Tsytovich, *Theory of Turbulent Plasma* (Consultants Bureau, New York, 1977), p. 210.
[31] S. Galtier and A. Bhattacharjee, Phys. Plasmas **10**, 3065 (2003).
[32] S. Ghosh, E. Siregar, D. A. Roberts, and C. A. Fitch, J. Geophys. Res. **99**, 2493 (1996).
[33] S. M. Mahajan and Z. Yoshida, Phys. Rev. Lett. **81**, 4863 (1998).
[34] O. Stawicki, S. P. Gary, and H. Li, J. Geophys. Res. **106**, 8273 (2001).
[35] H. Li, S. Gary, and O. Stawicki, Geophys. Res. Lett. **28**, 1347 (2001).
[36] L. Chen, Plasma Phys. **19**, 47 (1977).
[37] V. E. Zakharov, V. S. L'vov, and G. Falkovich, *Kolmogorov Spectra of Turbulence I* (Springer-Verlag, Berlin 1992).
[38] S. R. Cranmer and A. A. van Ballegooijen, Astrophys. J. **594**, 573 (2003).